

## Galactic Rotation

- From observations of the velocity of stars or gas around the Galactic center, the rotational velocity  $V$  can be determined as a function of the distance  $R$  from the Galactic center.
- We consider an object at distance  $R$  from the Galactic center which moves along a circular orbit in the Galactic plane, has a distance  $D$  from the Sun, and is located at a Galactic longitude  $l$ .
- We disregard the difference of velocities of the Sun and the LSR to get  $v_o = v_{LSR} = (v_o, 0)$  in this coordinate system.

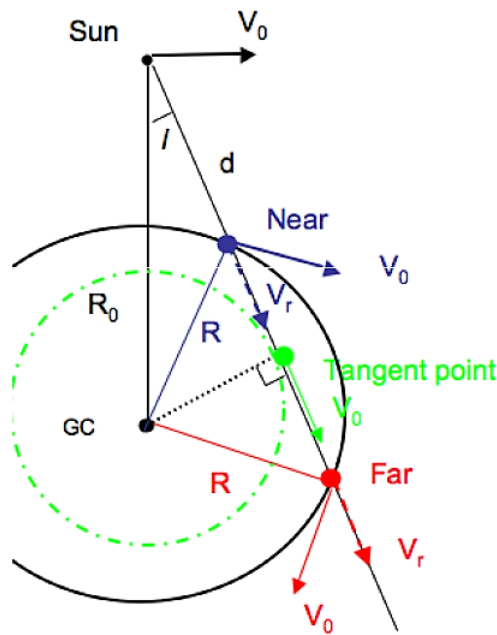


Figure 1: Tangent Point Method to Estimate Galactic Rotational Velocity

- Computation for the whole galactic rotation is done in two different ways :
  1. The Inner part(Tangent Point Method) ( $R < R_o$ )

**Where :**  $R_o = 8kpc$

- \* For this  $I$  and  $IV$  galactic quadrants are being considered.
- \* The radial velocity  $v_r$  along this line-of-sight for objects moving on circular orbits is a function of the distance  $d$ .

$$\begin{aligned} v_r &= v \cos(90 - \theta) - v_o \cos(90 - l) \\ v_r &= v \sin(\theta) - v_o \sin(l) \end{aligned} \quad (1)$$

\* Sine law (figure 2):

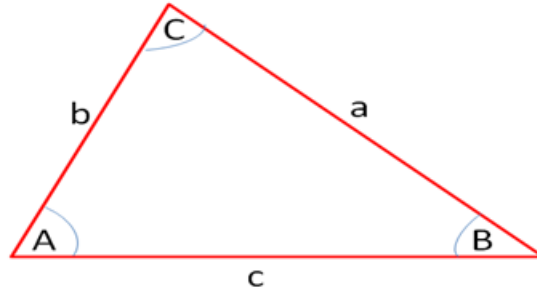


Figure 2: Sine rule triangle

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \quad (2)$$

\* Applying the sine law to find  $\theta$  in terms of  $R$

$$\frac{\sin(180 - \theta)}{R_o} = \frac{\sin(l)}{R} \quad (3)$$

$$\sin(\theta) = \sin\left(\frac{R_o}{R} \sin(l)\right)$$

\* Putting the value of  $\sin(\theta)$  in (1) :

$$v_r = v \sin(l) \frac{R_o}{R} - v_o \sin(l)$$

$$= \sin(l) \left[ v \frac{R_o}{R} - v_o \right] \quad (4)$$

$$v(R) = \left[ v_o + \frac{v_r}{\sin(l)} \right] \left( \frac{R}{R_o} \right)$$

\* The velocity  $v_r$  attains a maximum where the line-of-sight is tangent to the local orbit, thus its distance  $R$  from the Galactic center attains the minimum value  $R_{min}$ .

$$v(R) = \left[ v_o + \frac{v_{r,max}}{\sin(l)} \right] \left( \frac{R_{min}}{R_o} \right) \quad (5)$$

Where :  $R_{min} = R_o \sin(l)$

\* Above equation(5) gives the velocity of an object wrt. the inner distance  $R$  in galactic disc.

\* Figure(3) shows the velocity curve for different points.

## 2. The Outer part (Rotation Curve) ( $R > R_o$ )

**Where :**  $R_o = 8kpc$

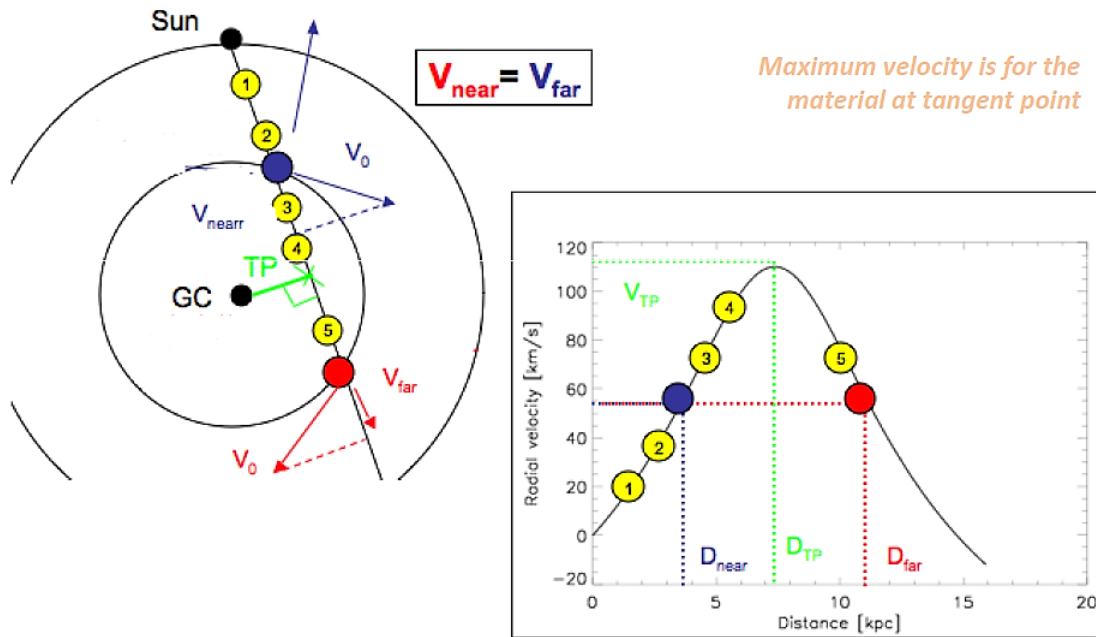


Figure 3: Tangent Point Method to Estimate Galactic Rotational Velocity

- \* For this *II* and *III* galactic quadrants are being considered.
- \* The tangent point method cannot be applied for  $R < R_0$  because for lines-of-sight, the radial velocity  $v_r$  attains no maximum.
- \* Measuring  $V(R)$  for  $R > R_0$  requires measuring  $v_r$  for objects whose distance can be determined directly e.g. Cepheids.
- \* Refer figure(5) for different calculation parameters.

$$\begin{aligned} v_r &= v_o \cos(l - 90) - v \cos(90 - \theta) \\ v_r &= v_o \sin(l) - v \sin(\theta) \end{aligned} \quad (6)$$

- \* Using sine rule we find the value of  $\sin(\theta)$  in terms of  $R$  and  $R_o$

$$\begin{aligned} \frac{\sin(\theta)}{R_o} &= \frac{\sin(l)}{R} \\ \sin(\theta) &= \sin\left(\frac{R_o}{R} \sin(l)\right) \end{aligned} \quad (7)$$

- \* Putting the value of  $\sin(\theta)$  in equation(6), we get

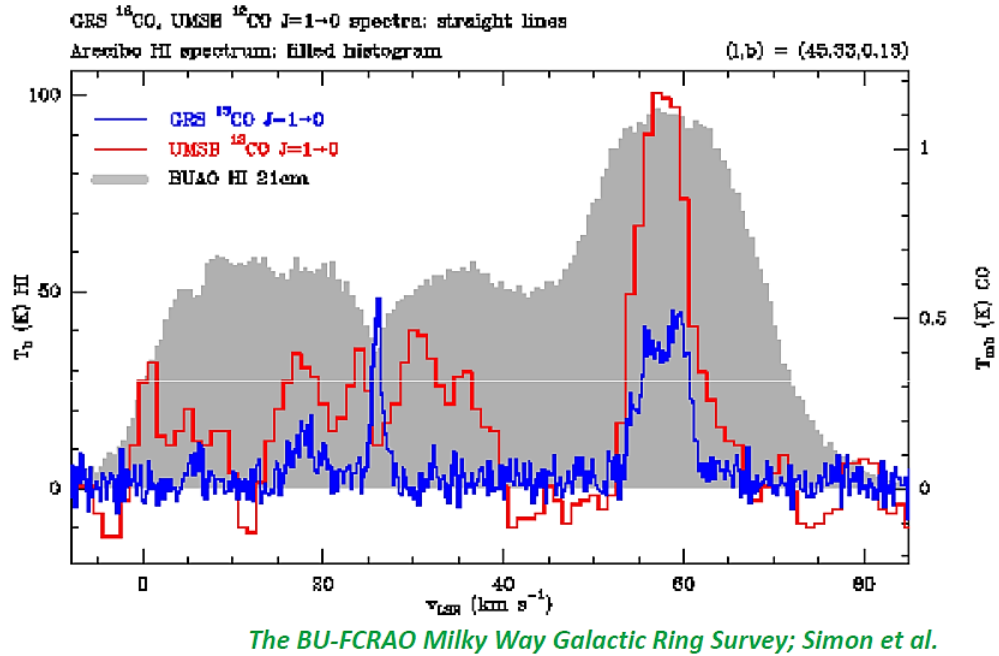


Figure 4: The figure shows  $HI_{21\text{cm}}$ , and  $CO^{12}$  and  $CO^{13}$  emission along a particular sightline through the Galactic plane. The emission with the largest  $v_{LSR}$  is likely to be close to the tangent point.

$$\begin{aligned}
 v_r &= v_o \sin(l) - v \sin(l) \frac{R_o}{R} \\
 &= \sin(l) \left[ v_o - v \frac{R_o}{R} \right] \\
 v(R) &= \left[ v_o - \frac{v_r}{\sin(l)} \right] \left( \frac{R}{R_o} \right)
 \end{aligned} \tag{8}$$

- \* Above equation(8) gives the velocity of an object wrt. the outer distance  $R$  in galactic disc.
- \* Figure(6) shows the Rotation curve for different orbital objects taken together after various surveys.

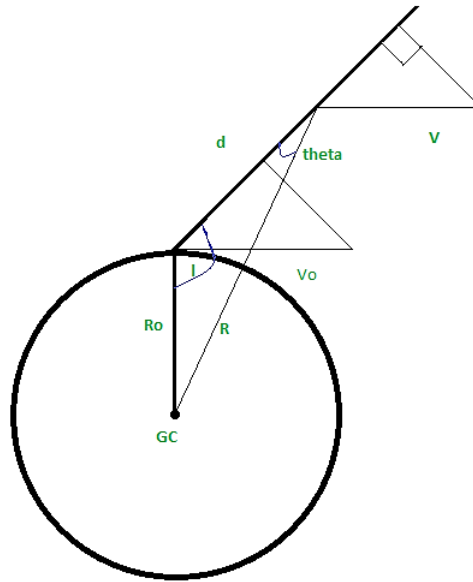


Figure 5: Rotation curve method

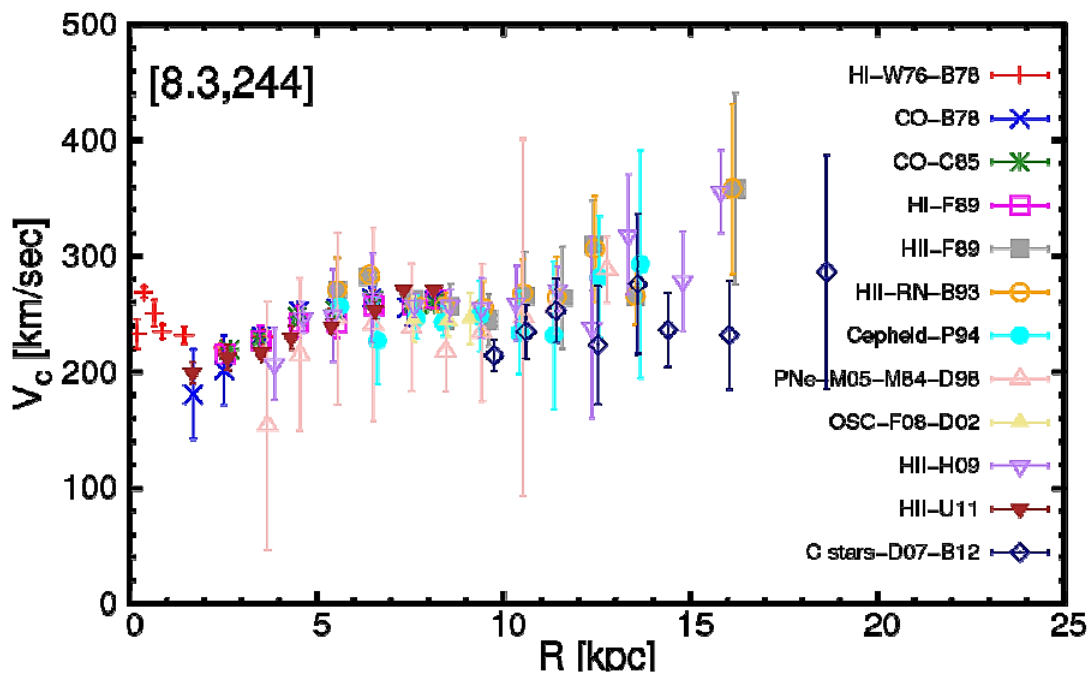


Figure 6: Milky Way Rotation Curve Bhattacharjee et al. (2014, *ApJ*)