Galactic Rotation Curve & Dark Matter

- Rotation Curve

The luminous matter distribution of our galaxy can be modelled as a circular disk with a bulge at the centre. The bulge has a roughly constant stellar density. Therefore, if radius of the bulge is R_b the mass enclosed within a radius r is given by,

$$M(r) = \frac{4}{3}\pi r^3 \rho_b (r < R_b) \tag{1}$$

So it follows that the rotational velocity in the bulge will be,

$$v^2(r) = \frac{GM(r)}{r} = \frac{4}{3}\pi \rho_b G r^2 \tag{2}$$

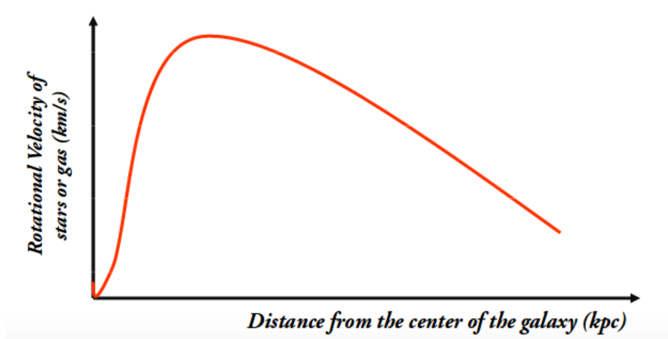
In the disk the stellar density declines exponentially. The velocity for disk density of rho_d, scale length l and disk radius of R_d is given by,

$$v^2(r) = \frac{4}{3}\pi \rho_d G r^2 = \frac{4}{3}\pi \rho_b \exp^{-\frac{r-R_b}{l}} G r^2 (R_b < r < R_d) \tag{3}$$

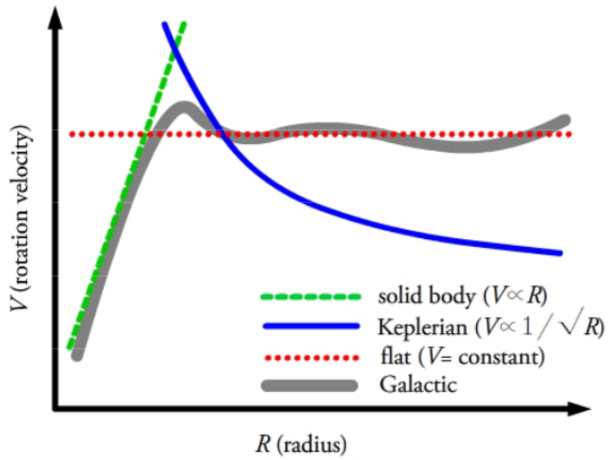
As r increases, exponential term begins to dominate in the above equation. Beyond a certain r, the mass remains almost constant. Therefore, we expect velocity profile for high r values as,

$$v^2(r) \alpha \sqrt{\frac{1}{r}} \tag{4}$$

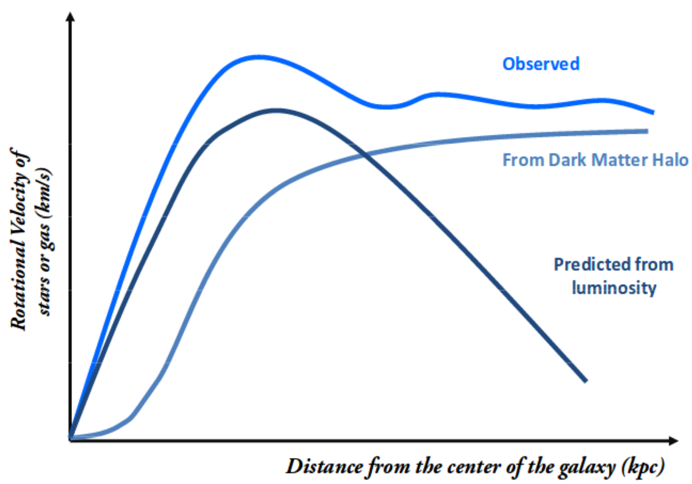
Thus, the expected rotation curve is as follows,



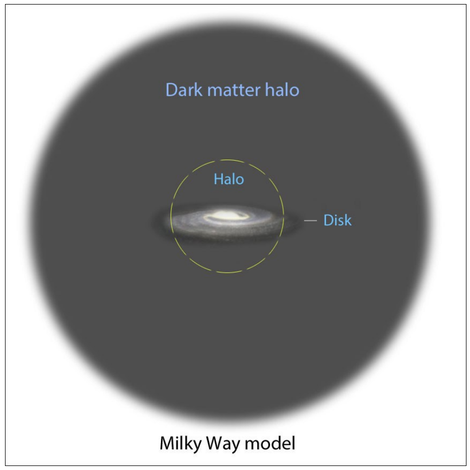
But strangely enough, we do not observe this. Instead, this is what we observe,



- Dark Matter Halo
  In order to account for the excess velocity, we introduce the concept of dark matter halo around our galaxy. This is how it's velocity profile should be,



The dark matter halo can be visualized as shown,



We can infer from all this that,
1. Missing mass is appproximately 10 times the luminous mass
2. Dark matter cannot be in the disk since otherwise it can disturb the long-term stability of the disk against the tidal forces.
3. Some fraction of it in the disk can be baryonic like dim stars (brown dwarfs, black dwarfs), and compact stellar remnants (neutron stars, black holes).
4. Dark matter halo emits no light and thus has a completely unknown spatial distribution. Density profile in the halo goes as $\rho \alpha r^{-2}$

Mass distribution of the Milky Way is found to be as given below,

- **Stellar mass ~ 10^{10} $M_\odot$** *(all sky surveys, point source counting)*
- **ISM mass ~ 10^9 - $10^{10} M_\odot$** *(H I 21 cm surveys)*
- **Luminous mass (Stellar + ISM) ~ 10^{10} $M_\odot$**
- **Dynamical mass ~ 10^{12} $M_\odot$**
  dominated by dark matter (from rotation curve)