

Gravitational Lensing by Clusters, Cluster mass from lensing

Gravitational Lensing by Clusters

Gravitational lensing is an effect of Einstein's theory of general relativity. The gravitational field of a massive object will extend far into space, and cause light rays passing close to that object (and thus through its gravitational field) to be bent and refocused somewhere else.

The first evidence for Einstein's general theory of relativity, gravitational lensing was obtained from the observation performed by Sir Arthur Eddington during a total solar eclipse. He observed the Haydes cluster behind the sun and found an offset in its position.

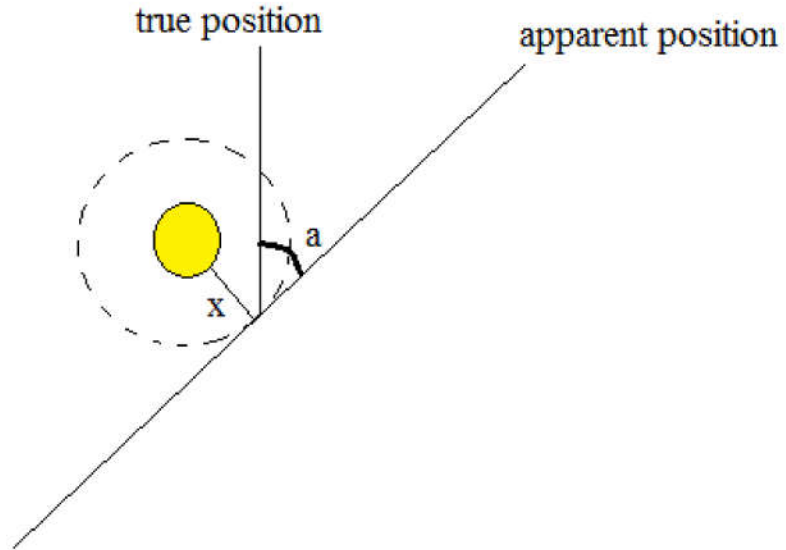


fig1: A ray of light passing near the sun

$$\alpha = \frac{4GM}{c^2} \frac{1}{x} = \frac{2r_s}{x} \quad (1)$$

For Eddington's experiment, the light from Haydes was grazing the surface of the sun. So putting the values $x = R_{sun} \approx 10^5 r_s$, where r_s is Schwarzschild radius, we get

$$\alpha = \frac{2r_s}{10^5 r_s} = 2 \times 10^{-5} \text{ radians} \approx 1.8'' \quad (2)$$

Clusters of galaxies are very massive, and their gravitational fields will deflect the light from background galaxies, producing deflections, distortions and multiple images. A very good example is the distorted structures in Abell 370 (shown in fig 2). The redshift, z of this galaxy cluster is 0.375. The lensed image which has been marked has a different redshift, which gives a clear idea that it is a background object.

Cluster Mass from Lensing

Lets consider a gravitation lensing process as illustrated below in fig 3. The light ray from source S pass near the massive object L(which act as the lens) at a distance x from the center of L . An observer at O will see two images S_1 and S_2 of the source due to lensing.

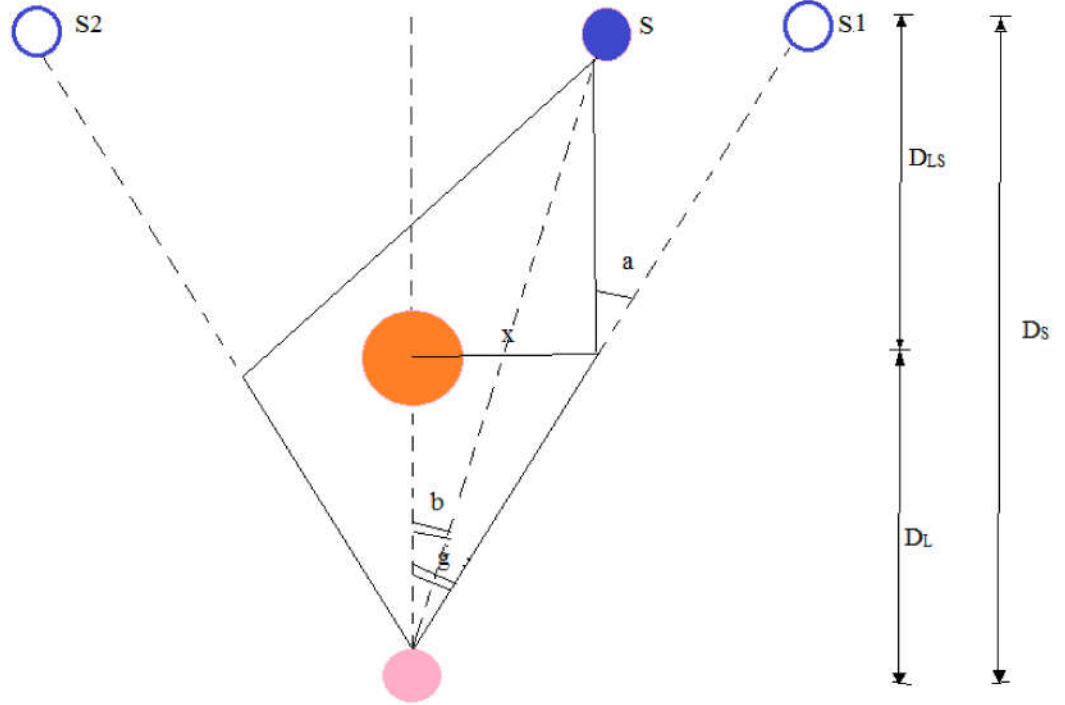


fig 3: Gravitational lensing

From the above figure,

$$\theta D_S = \beta D_S + \alpha D_{LS} \quad (3)$$

$$\theta - \beta = \frac{\alpha D_{LS}}{D_S} \Rightarrow \beta = \theta - \frac{\alpha D_{LS}}{D_S} \quad (4)$$

β (angle between source and line of sight) and θ (angle between S_1 and line of sight) are direct observable, whereas α is not a direct observable but it is related to mass as

$$\alpha = \frac{4GM}{c^2} \frac{1}{x} \quad (5)$$

So, (4) can be rewritten as,

$$\beta = \theta - \frac{4GM}{c^2 \theta} \left(\frac{D_{LS}}{D_L D_S} \right) \quad (6)$$

When the source is just behind the lensing object, then $\beta=0$ and hence we get,

$$\theta = \left[\frac{4GM}{c^2} \left(\frac{D_{LS}}{D_L D_S} \right) \right]^{\frac{1}{2}} = \theta_E \quad (7)$$

From (7) and (6),

$$\beta = \theta - \frac{\theta_E^2}{\theta} \Rightarrow \theta^2 - \beta\theta - \theta_E^2 = 0 \quad (8)$$

$$\theta_{\pm} = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2} \quad (9)$$

θ has two solutions which suggest that there are two images(in opposite directions). For a spherical mass distribution, the image for a source directly behind the lens is an Einstein ring. In case of a spheroidal distribution we obtain a Einstein's cross. Irregular structure such as clusters form lensed images in the shapes of arcs, arclets, shear. The distances used in these expressions are angular diameter distances calculated from the redshifts. The distance between two points at redshift z_1 and z_2 is given by $r(z_1, z_2)$:

$$r(z_1, z_2) = r(z_2) - r(z_1) = \frac{c}{H_0} \int_{z_1}^{z_2} \frac{dz}{(\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0})} \quad (10)$$