Accretion Disk/Accretion Disk Temperature

- **Accretion Disk**
  Let's assume the disk to be geometrically thin optically thick disk(optical depth $\tilde{-}$ 1000).

  The distance 'R' and the radius of the disk is comparable i.e. $R\tilde{-}r$.

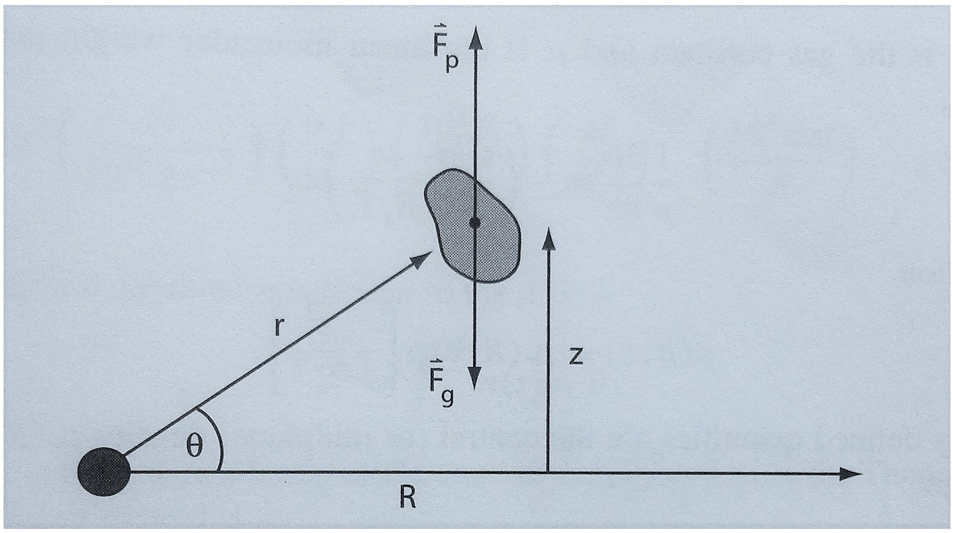

Figure 1: Assuming the gas to be in hydrostatic equilibrium. Gravity acting along the central portion making an angle θ with the plane of the disk.

$$F_z = F * sin(\theta) \tag{1}$$

$\sin(\theta) = \frac{z}{r}$ where: $r\tilde{-}R$

$$F_z = \frac{GMz}{R^3} \tag{2}$$

Now from hydrostatic equilibrium case:

$$\frac{dP}{dz} = -\rho g \tag{3}$$

From Sound equation in isothermal case:

$$C_s^2 = \sqrt{\frac{\gamma P}{\rho}} \tag{4}$$

Since, $\gamma = 1$ for isothermal case $C_s^2 = \frac{P}{\rho}$

Differentiating the above P equation wrt. z

$$\frac{d\rho}{\rho} = -\frac{GM \int zdz}{R^3 C_s^2} \tag{5}$$

$$\int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\frac{\omega^2}{C_s^2} \int_0^z zdz \tag{6}$$

Solving equation (6) we finally get the solution of the equation as:

$$\rho = \rho_o e^{\frac{-z^2}{H^2}} \tag{7}$$

Where $H = \frac{2C_s^2}{\omega^2}$
H is the scale height.

Considering the scale height equation for the disk geometry prediction as:

$$\frac{H^2}{R^2} = \frac{2C_s^2}{v_c^2} \tag{8}$$

This gives a sense that for an thin geometry ($\frac{H}{R}$) the accretion of the gas happens at an supersonic speed around the central engine(SMBH).
The angular momentum, $L = \sqrt{GMR}$ of the rotating gas.

- **Accretion Disk Temperature**
  We have the equation for potential energy as:

$$dU = \frac{GM}{R^2} dmdR \tag{9}$$

Let suppose the luminosity produced due to this accretion is 0.5 times the rate of change of potential energy from above equation.

$$L = \frac{1}{2}[\frac{GM}{R^2}(\frac{dm}{dt})dR] \tag{10}$$

Since we are considering the disk to be optically thick disk so we can take the luminosity/area value from black body luminosity equation.

$$\frac{L}{2 * 2\pi RdR} = \sigma T^4 \tag{11}$$

In the above equation the extra 2 term is because of the fact that the disk radiates from both sides.
Solving the equation (10),(11) we finally get,

$$T = [\frac{GM}{R^3}\frac{dm}{dt}\frac{1}{8\pi}]^{\frac{1}{4}} \tag{12}$$

The term $\frac{dm}{dt}$ is the mass accretion rate.
By doing more rigorous calculations the temperature of the disk can be found out as:

$$T = [\frac{GM\frac{dm}{dt}}{8\pi R^3\sigma}[1-\sqrt{\frac{R_{ISO}}{R}}]]^{\frac{1}{4}} \tag{13}$$

$$T = 6 * 10^5[\frac{M}{10^8 M_o}]^{\frac{1}{4}}[\frac{R}{R_{ISO}}]^{-\frac{3}{4}}[\frac{(\frac{dm}{dt})}{(\frac{dm}{dt})_{edd}}]^{\frac{1}{4}}K \tag{14}$$

By putting the proper values of different identities in the equation (14) we finally being able to say that the temperature across the central engine in AGNs are of the order 10^5.

- **Notes:**

   i. As the mass accretion happens inside the transfer of angular momentum happens outwards.

   ii. The outer portion of the SMBH cannot remains as the thin disk but get fluffed as more and more amount of momentum transferred outwards(Shown in figure (2)).

   iii. The effective temperature of the disk is proportional to: $T_{eff} = M^{-\frac{1}{2}}$, M is the mass of the central engine (eg. SMBH, Neutron star, White Dwarf ,LMXB).

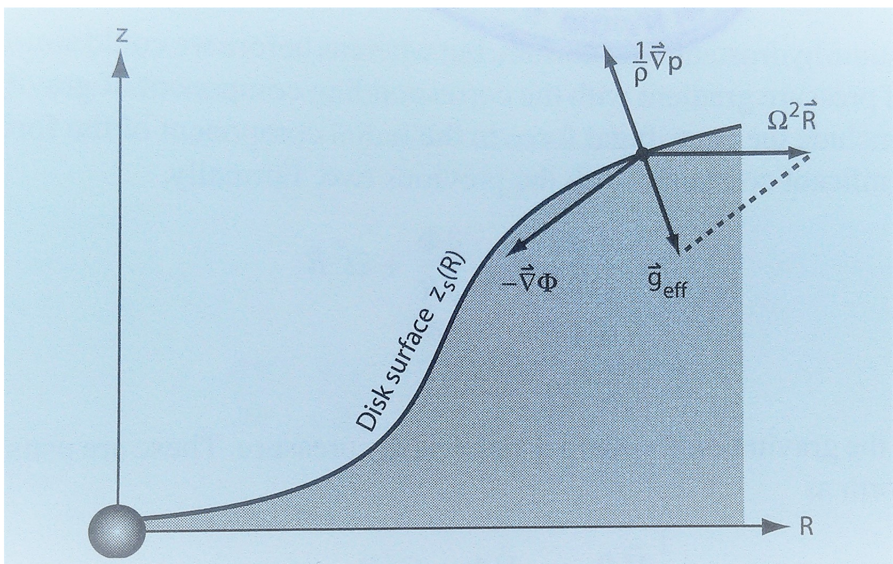

Figure 2: When the amount of angular momentum increases outwards the disk start to expand in the vertical direction as to maintain the circular orbit below an certain Keplerian value.