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Tully–Fisher Relation

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Tully–Fisher Relation

Cepheid variable stars are the primary means by which distances are measured over the local volume of space. However, beyond about 20 megaparsecs (Mpc) Cepheids become too faint, even for Hubble Space Telescope, and so alternative means of measuring distances are needed. One of the more popular secondary methods makes use of the strong correlation between the luminosity of SPIRAL GALAXIES and their rotational velocities. This is known as the Tully–Fisher relation.

The Tully–Fisher relation has become one of the most widely used methods of measuring extragalactic distances since spiral galaxies are relatively common and contain young, massive stars. As a result, they also contain Cepheid variables, making the calibration of the Tully–Fisher relation using nearby systems relatively straightforward. In applying this technique the rotational velocity serves as a predictor of the luminosity, or absolute magnitude, of the galaxy. The distance is then calculated from the distance modulus, the difference between the apparent magnitude and the predicted absolute magnitude of the galaxy. The Tully–Fisher relation provides an opportunity to step from the local calibrating galaxies to the smooth Hubble expansion in a single step. Consequently, this technique is extremely valuable for the mapping of large-scale structure, the Hubble flow and any associated peculiar (i.e. non-expansion) velocities.

The origin of the Tully–Fisher relation remains somewhat uncertain but arises from the correlation between mass and luminosity. A spiral galaxy rotates under the influence of its own gravity and so the rotational speed is connected with its mass. A more massive galaxy also contains a larger number of stars and is therefore more luminous. The result is a correlation between the luminosity and the rotational speed for spiral galaxies, the Tully–Fisher relation. There are a few complications to this simple explanation. First, the total luminosity of a particular spiral galaxy is a reflection of its star formation history. This might be expected to vary from galaxy to galaxy. Second, the fact that spiral galaxies have considerable amounts of 'DARK MATTER' means that the mass of a galaxy is not necessarily well represented by the amount of mass contained within its population of stars. That is, the ratio of mass to light for spiral galaxies might be expected to vary at a given total mass. The fact that the Tully–Fisher relations show relatively small scatter indicates that these variations are small and this in turn provides a strong constraint on models for the formation and evolution of spiral galaxies. While these details are still uncertain we can still measure galaxy distances through an empirical calibration of this relation.

Observational data for the Tully–Fisher relation

The necessary observational data consist of apparent magnitudes, corrected for Galactic and internal extinction by dust, and some measure of rotational velocities, corrected for projection effects. Usually, the rotational velocity is measured via the Doppler broadening of the H I 21 cm line, since spiral galaxies are easily detected in this spectral line using RADIO TELESCOPES. However, for galaxies with REDSHIFTS of only a few thousand km s^{-1}, the 21 cm line is shifted into heavily used regions of the radio spectrum, resulting in severe interference from terrestrial sources (radars, cell phones, etc). In addition, at this wavelength the resolution of even the largest single-dish radio telescopes is only a few arcminutes and so for redshifts beyond about 5000 km s^{-1} the detected signal can be confused with that from neighboring galaxies. This has prompted interest in using optical emission lines such as Hα to measure the rotational speed of spiral galaxies.

Early application of the Tully–Fisher relations used photographic photometry, but today CCDs are the detectors of choice at optical wavelengths owing to their superior performance. The redder Johnson bandpasses (R and I) are usually used because of their reduced sensitivity to dust extinction and because the spectral energy distributions of spiral galaxies are dominated by late-type giant stars at these wavelengths. Thus, these measurements are less sensitive to variations in the star formation history and dust content of the individual galaxies. These advantages continue into the near-infrared H and K bands. However, the night sky at these wavelengths is as much as a factor of 10^4 brighter than at optical wavelengths, making accurate surface photometry at faint levels extremely difficult.

A typical procedure for measuring the magnitude of a galaxy from a calibrated CCD image begins by identifying and masking contaminating regions from foreground stars, background galaxies and cosmetic defects. The surface brightness profile of the galaxy is constructed by binning pixels as a function of radius. The integrated magnitude within some surface brightness level can then be computed and this measurement extrapolated to infinite radius using some model, such as an exponential, to produce a total magnitude for the galaxy. Ellipses are often fitted to the isophotes of the galaxy to yield the best-fit axial ratio of the inclined disk as a function of radius and surface brightness. The axial ratio can then be used to estimate the inclination of the galaxy.

There are several advantages to using the Doppler-broadened 21 cm line of H I as the measure of the rotation of late-type galaxies. First, spiral galaxies are H I rich so the line is extremely strong. Second, the H I within these systems has an extended distribution such that the outer, flat portion of the rotation curve is well sampled, providing an accurate measurement of the maximum rotational velocity. However, low angular resolution and interference become significant limitations at redshifts of $\sim$10 000 km s^{-1}, as described above. The use of the Hα emission line avoids these issues. In this case, the slit of a spectrograph is positioned to lie along the long axis of a DISK GALAXY such that the resulting spectrum will sample the velocity as a function of radius for the galaxy. However, since the Hα emission line is produced within regions of active STAR FORMATION, the outer regions of spiral galaxies may not have enough Hα emission to be detected. This can result in the outer, flat portion of the rotation curve of lower-luminosity galaxies not being adequately sampled since these systems generally have slowly rising rotation curves. Thus, 21 cm measurements are usually the best choice for nearby galaxies and Hα measurements are optimal for redshifts beyond about 10 000 km s^{-1}. For redshifts beyond about 0.2 (60 000 km s^{-1}) the Hα line becomes contaminated by increasingly strong emission lines from the night sky (mostly due to OH). This has prompted investigations into whether other lines such as [O II] (λ 3727 Å) and [O III] (λ 5007 Å) could be used such that the Tully–Fisher relation could be applied at larger distances. The preliminary results appear to be very promising and efforts are underway to use the Tully–Fisher relations to examine the structural and evolutionary properties of GALAXIES AT HIGH REDSHIFTS.

Inclination corrections to the observational data

Some method of measuring galaxy inclinations is required in order to correct the magnitudes for internal extinction and the observed rotation for projection. Assuming that all spiral galaxies can be adequately represented as an oblate spheroid with the same intrinsic flattening, the inclination can be calculated from the projected shape of the disk. Specifically,

$$\cos^2 i = \frac{(b/a)^2 - \alpha^2}{1 - \alpha^2}$$

where i is the inclination, b/a is the observed axial ratio of the best-fitting ellipse and α is the intrinsic axial ratio for an edge-on system. Complications arise from the fact that some galaxies do not have perfectly circular isophotes at face-on orientation. As a result, most investigations restrict the sample to those systems whose inclinations are at least 30°–45° in order to minimize the resulting uncertainties on the deprojected rotational velocities.

The parameter of choice for predicting the luminosity of a disk galaxy is $V_{\rm max}$, the maximum amplitude of the rotation curve. This is especially convenient given that most galaxies have flat rotation curves over much of their extent. Tully and Fisher advocated using the Doppler width of the 21 cm line profile measured at 20% of peak intensity (W_{20}), corrected for inclination. That is, $W_{\rm R}^{\rm i} = W_{20}/\sin(i) \sim 2V_{\rm max}$. However, if the Tully–Fisher relation is to be applied to low-luminosity spiral and irregular galaxies a more complicated correction is required in order to account for the effects of internal turbulence and subtle variations in line-profile shape. However, for $W_{\rm R}^{\rm i} >$ 150 km s^{-1} these corrections are negligible. When optical spectra are used $V_{\rm max}$ can be measured directly from the rotation curve.

Historically, the internal extinction with disk galaxies has proven to be a difficult problem. However, the advent of CCDs and infrared detectors has allowed accurate photometry of galaxies over a broad range in wavelength enabling the amount of extinction to be statistically estimated from the amount of reddening. The extinction corrections to the magnitudes are often expressed in terms of the observed axial ratio. Specifically, $A_\lambda^{\rm i} = \gamma_\lambda \log(a/b)$, where λ is the wavelength of the observation in the standard bandpasses and a/b is the observed axial ratio. The dependence of this correction on the luminosity of the galaxy, due to variations in dust content, etc, can also be described by expressing γ_λ in terms of the projection-corrected rotational velocity (W_R^i). A representative result is

$$\gamma_B = 1.57 + 2.75(\log W_{\rm R}^{\rm i} - 2.5)$$

$$\gamma_R = 1.15 + 1.88(\log W_{\rm R}^{\rm i} - 2.5)$$

$$\gamma_I = 0.92 + 1.63(\log W_{\rm R}^{\rm i} - 2.5)$$

$$\gamma_H = 0.43 + 0.76(\log W_{\rm R}^{\rm i} - 2.5)$$

where the subscripts on γ represent the standard bandpasses. Here, $W_{\rm R}^{\rm i} \sim 2V_{\rm max}$ and $V_{\rm max}$ is the amplitude of maximum rotation within a galaxy.

The apparent magnitudes also require correction for Galactic extinction. In the past, this has been a difficult problem. However, there now exist extensive maps of both the neutral hydrogen gas (H I) and the distribution of dust (IRAS 100 μm) within the Galaxy. The good correspondence between these two maps indicates that the H I gas and dust trace each other. The amounts of the extinction which correspond to these two components have been calibrated from detailed, multicolor observations of distant stars. As a result, the Galactic extinction for any galaxy can be estimated given its position on these maps. As the Galactic latitude falls below about 25–30°, the corrections for Galactic extinction become large with correspondingly large uncertainties. Therefore, most investigators limit the application of the Tully–Fisher relation to galaxies above this latitude.

Absolute calibration of the Tully–Fisher relation using Cepheids

Ground-based Cepheid surveys are extremely difficult because the Cepheids in all but the nearby Local Group galaxies are very faint and subject to significant contamination by neighboring stars. However, the advent of Hubble Space Telescope has allowed the detection of Cepheids within galaxies out to about 20 Mpc in distance and enabled the number of calibrating systems to be significantly increased. A large effort to obtain Cepheid distances to a number of nearby galaxies, known as the Hubble Key Project of Extragalactic Distances, and a similar project to obtain Cepheid distances to galaxies containing type Ia supernovae, has brought the number of galaxies suitable for calibrating the Tully–Fisher relation to 30 systems. Figure 1 shows the resulting Tully–Fisher relations for this calibration sample. A least-squares fit to these data gives the following relations:

$$M_B^{\rm b,i} = -7.41(\log W_{\rm R}^{\rm i} - 2.5) - 20.04 \pm 0.04 \quad (0.22)$$

$$M_R^{\rm b,i} = -8.09(\log W_{\rm R}^{\rm i} - 2.5) - 21.05 \pm 0.04 \quad (0.19)$$

$$M_I^{\rm b,i} = -8.55(\log W_{\rm R}^{\rm i} - 2.5) - 21.51 \pm 0.04 \quad (0.22)$$

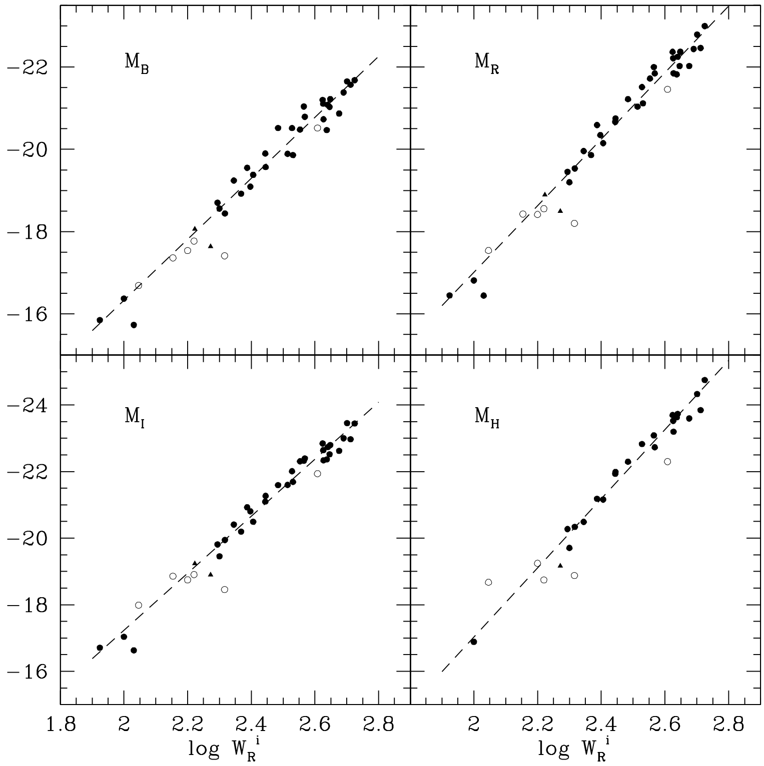

Figure 1. *B*, *R*, *I* and *H* band calibrations of the Tully–Fisher relation. Solid circles are galaxies with distances measured using Cepheids, solid triangles are galaxies with distances estimated via surface brightness fluctuation measurements within dE companions and open circles are systems thought to be group members with at least one galaxy with a Cepheid distance, and therefore thought to be at a similar distance. The dashed line is a least-squares fit to the solid points in each panel.

$$M_H^{\mathrm{b,i}} = -10.39(\log W_{\mathrm{R}}^{\mathrm{i}} - 2.5) - 22.22 \pm 0.08 \quad (0.30)$$

where the superscripts b and i indicate Galactic extinction and internal extinction corrections, respectively. The zero points give the absolute magnitude for galaxies with $\log W_{\mathrm{R}}^{\mathrm{i}} = 2.5$, or $V_{\mathrm{max}} = 160$ km s^{-1}. In all bandpasses, the random uncertainty in the zero point is negligible compared with the remaining systematic uncertainty in the absolute calibration of the Cepheids themselves (∼0.13 mag, or 7% in distance). The rms dispersions of the relations are given in parentheses. Given these calibrations, the absolute magnitude of any spiral galaxy can be predicted from its rotational velocity and its distance estimated from its measured apparent magnitude.

Application of the Tully–Fisher relation to field galaxies

The Tully–Fisher relations have been used to estimate the distances to several hundred spiral galaxies within an expansion velocity of about 8000 km s^{-1}. An example of the resulting 'Hubble flow' is shown in figure 2. Since the velocities are a combination of both the expansion and any local peculiar velocities induced by gravity, the deviations in velocity can be correlated with position on the sky to produce maps of the deviations from the mean expansion field. The primary features seen in such maps are (1) local streaming of ∼100 km s^{-1} associated with the non-uniform distribution of matter within 10 Mpc of the Local Group, (2) retarded expansion velocities of several hundred km s^{-1} near the Virgo cluster of galaxies and (3) a large-scale streaming of about 400 km s^{-1} towards a 'Great Attractor' located at a distance of about 60 Mpc in the direction of the Hydra–Centaurus cluster complex. These peculiar velocities are usually assumed to result from the gravitational accelerations produced by the irregularities in the large-scale distribution of mass. With this assumption, various research groups have used these 'flow maps' to constrain the mean mass density of the universe (Ω) as well as to examine the degree to which the distribution of galaxies follows the distribution of mass. These investigations appear to favor low values of Ω (∼0.2) and also imply that mass is generally traced by the distribution of galaxies.

Distant clusters and the Hubble constant

The errors in the distances of individual galaxies measured using the Tully–Fisher relations are about 15% when using high-quality observational data. As a result, the absolute errors can become large for distant galaxies. Since there are still significant peculiar velocities out to velocities of

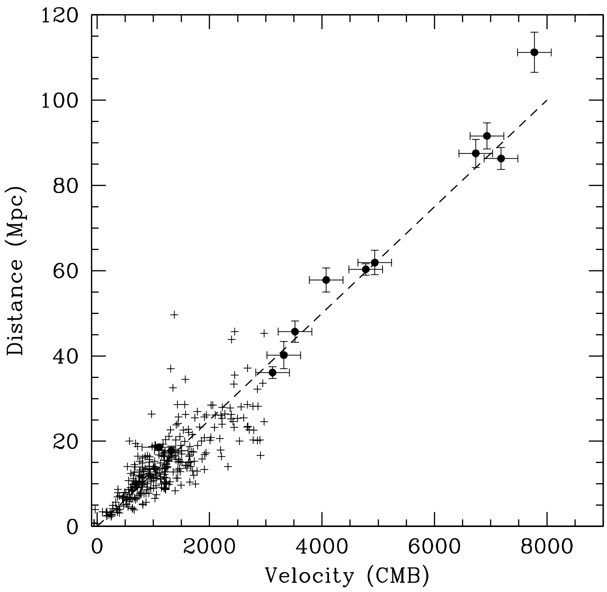

Figure 2. The velocity–distance relation, or 'Hubble flow', illustrating the expansion of the universe. The plus signs show the distances to individual, nearby galaxies measured using the Tully–Fisher relation. Since the errors are fractional (i.e. logarithmic), the absolute errors increase with distance and result in a cone-like spread of the data. The solid points with error bars represent the average distance and velocity for clusters. The cluster samples typically contain 15–20 galaxies, correspondingly reducing the errors. The dashed line shows a Hubble constant ($H_0 = V/D$) of 80 km s^{-1} Mpc^{-1}.

about 5000 km s^{-1}, large samples are needed at these distances in order to accurately sample the Hubble flow and map the distribution of peculiar velocities. Recent Tully–Fisher surveys using cluster samples have been used to measure the mean distances for a number of clusters over this volume. The mean distances and velocities of these clusters are shown in figure 2 along with those of the nearby field galaxies. Together, these data yield an estimated value of the HUBBLE CONSTANT of 77 ± 7 km s^{-1} Mpc^{-1}.

The inverse of the Hubble constant is related to the age of the universe. For a universe with zero density H^{-1} is equal to the age of the universe. If the universe has the 'critical density', i.e. $\Omega = 1$, then its age is $(2/3)H^{-1}$. At the present time, a typical estimate for the age of globular cluster stars is 14 ± 2 billion years. For comparison, the value of the Hubble constant given above corresponds to ages of 12.7 ± 1 and 8.5 ± 0.7 billion years for $\Omega = 0$ and $\Omega = 1.0$, respectively. At present, it appears that a cosmologically flat, critical universe is inconsistent with the observational data. This discrepancy has helped to generate interest in cosmological models with a non-zero COSMOLOGICAL CONSTANT Λ and, in particular, in cosmologically flat models with non-zero Λ. A cosmological constant is one way of reconciling these disparate estimates of the age of the universe.

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